The basic Mach card is a folded white card or piece of paper. It is included with this paper as Card 1. This card is illuminated by a light source that is off to the side of the card. The observer of the illusion closes one of his/her eyes, and looks at the middle of the card with the other eye at about a 45 degree angle from the horizontal plane. This setup is shown in the following pictures:

While looking at the card, the observer makes the perceived three dimensional orientation of the card reverse. That is, he/she makes him/herself perceive the card as if the nearer parts of the card were far away, and the farther parts of the card were near. The card then appears as an inside-corner facing the observer.

Because the card is illuminated from the side, the side facing the light source is brighter than the side facing away from the light source. The main part of the illusion is that once the observer makes the card reverse, he/she perceives the darker side as being
even darker, and the lighter side as being even lighter. In fact, on the darker side, the
color of the card itself appears to become gray instead of white.

One of the processes that plays a role in this illusion is the perceptual reversing of
depth. The other (and more interesting) process is how the colors of the card appear to
change, even though the actual color and illumination do not. I will first take a brief look
at the first process, and make some improvements on the illusion regarding that process.
Then, for most of the paper, I will examine the second process.

Perceptually reversing a 2D picture of a 3D object is relatively easy compared to
reversing an actual 3D object. Consider how easy it is to reverse the following picture of
a Mach card:

![Mach Card Image]

The reason this 2D image is so much easier to reverse than the actual card is that there are
fewer depth cues in this image than there are in the actual card. The fewer depth cues
there are, the less information there is that can be used to determine the 3D shape of the
object. This means the image is more ambiguous; the more ambiguity there is, the easier
the object is to reverse. To improve the Mach card illusion, I tried to make the card easier
to reverse by eliminating depth cues. The cues I was able to eliminate or minimize are
binocular cues, accommodation, and perspective.

Binocular depth cues are easy to eliminate — all the observer has to do is close
one of his/her eyes, and look with just one eye.

A first approach to minimize the affects of the accommodation cue might be for
the observer to not focus his/her eye anywhere on the card. However, this makes the card
blurry. Since the card is the very thing we want to analyze, this is a bad side effect to have. A better solution is for the observer to focus on one point on the card while he/she is reversing the card. In this case, the lens accommodates just one depth. Because it never accommodates any other depths, it supplies very little information. In addition, the card does not appear blurry in the local area that the observer is fixating. And because the card is relatively small, it is not too blurry in general.

Another cue we can eliminate is perspective. By cutting the sides of the card farthest away from the observer, we make the 2D image of these sides appear parallel to the sides closest to the observer. This makes the image of the card easier to reverse. These cuts are made in Card 2.

The most interesting part of the Mach card illusion is that the colors of the two sides of the card appear to change. As stated before, the dark side becomes darker and the light side becomes lighter. This is due to brightness constancy.

Brightness constancy refers to the fact that no matter what illumination one perceives an object in, the object appears to have the same brightness. One of the key points of brightness constancy is that in order to tell the color of an achromatic color in some illumination, an observer needs another color in the same illumination to compare it too. This was shown in an experiment by A. Gelb. In his experiment, a piece of black paper was illuminated in an otherwise completely dark room. Depending on the intensity of the illumination, subjects reported the color of the paper to be anywhere from white to black. However, when a white piece of paper was put in front of the black paper, subjects always perceived the white paper as white and the black paper as black.(Beardslee and Wertheimer 226)
This result, however, does not say anything quantitative. Hans Wallach was able to show that the ratios of luminances of colors is what accounts for constancy (Beardslee and Wertheimer 242). He did this by setting up projectors to make two somewhat distant configurations of light in an otherwise dark room. Each configuration contained two parts: an inner circle, and a ring around the inner circle. The brightness of the inner circle and the ring could be varied independently. Subjects were presented with these two configurations, where the inner circles and rings were of different brightness. One configuration acted as a “reference” and could not be changed. In the other configuration, the intensity of the ring remained constant, but the intensity of the circle could be varied (this was also done the other way around — where the ring could be varied, but the circle was left constant). The intensity of the ring in this variable configuration was different than the intensity of the ring in the reference configuration. The task of the subjects was to set the intensity of the circle in the variable configuration such that the color of the circle appeared to be the same as the color of the circle in the reference configuration. This is depicted below:
Wallach showed that subjects perceived the colors of the two inner circles as the same when the ratio of the intensity of the inner circle to the intensity of the outer ring was the same for each circle-ring pair. (Beardslee and Wertheimer 225 - 242)

Another fact necessary to explain the Mach card illusion is that “a luminance difference at a corner (or dihedral angle) will often be interpreted as an illumination rather than a lightness change” (Rock 210). An experiment to test this exposes subjects to a card folded at a right angle with two tabs on it, as in the picture below:

![Card Diagram]

Although it isn’t shown in the picture, the two tabs were actually trapezoidal. This was so that if a subject viewed this card with just one eye from a special viewpoint, the tab in the horizontal plane would appear to be in the vertical plane, and the tab in the vertical plane would appear to be in the horizontal plane. Subjects saw the card either with both eyes, or with just one eye from the special viewpoint. The card was illuminated so that everything in the horizontal plane got a lot of illumination, and everything in the vertical plane got very little illumination. The subjects’ task was to report the colors of the tabs.

Subjects that could see the card with both eyes reported that the tab in the vertical plane (the white tab) was white, and the tab in the horizontal plane (the black tab) was black. However, subjects that saw the card with only one eye from the special viewpoint reported that the vertical tab (the white tab) was black, and the horizontal tab (the black
tab) was white. (Remember that these subjects perceived the vertical tab to be in the horizontal plane and the horizontal tab to be in the vertical plane). Since the only thing that varied between the subjects was which planes they perceived the tabs to be in, the subjects must have been attributing the difference in luminance between the two tabs to the difference in illumination of the two planes. (Rock 246 - 247)

The results of these two experiments can be applied to explain the Mach card. The Mach card is a dihedral angle, so an observer is likely to attribute the difference in luminance between the dark and light sides of the card to illumination. Define a unit of illumination to be the illumination of the dark side of the card. Then the illumination of the dark side is 1, and that of the light side is $I$, where $I > 1$. Also, because the card is really the same color everywhere, the reflectance of both the dark and light sides is 1, where we define a unit of reflectance to be the reflectance of the card. Since luminance equals illumination times reflectance, we get that the luminance of the dark side is 1, and that of the right side is $I$.

When the card is perceptually reversed, the perceived illuminations of the dark and light sides are switched. This is because the observer is able to see the shadow of the card (and other nearby objects), and thus is able to infer that the location of the light source did not change. Since the sides of the card that face toward and away from the light source have switched roles, the illuminations of the sides switch. Thus, the illumination of the dark side becomes $I$ and that of the light side becomes 1.

Now, the actual luminance of each side of the card obviously does not change, so the luminances remain 1 for the dark side and $I$ for the light side. Therefore, the perceived reflection of the dark side must be $1/I$ and that of the light side must be $I$. Since $I > 1$, this
means the color of the dark side is darker than it was originally, and the color of the light side is lighter than it was originally. This is exactly the effect that happens.

Note that according to Wallach, when we compare colors under different illuminations, we should actually be looking at the ratios of the colors to other, constant colors. In the above computations, we did not use any such ratios. The argument above is still correct, though, since the perceived illumination changes only for the card, and the illumination and luminance of everything surrounding the card does not change. This means that when the observer compares ratios relating luminance of a color to luminance a surrounding color, the part of the ratios from the surrounding color always cancels out. To see this, let the luminance of a side of the card be \( C \), the luminance of a surrounding color be \( S \), and the luminance of a test color be \( T \). Then when the observer calculates the color of the side of the card, he/she compares the ratio \( C / S \) to the ratio \( T / S \), and gets some relation \( R \). (For example, if the observer finds that the test color matches the perceived color, then \( R \) would be equality). Note that

\[
\frac{C}{S} \frac{R}{T} \frac{T}{S} \iff C R T.
\]

Therefore, when we do the computations for the Mach card, we can ignore the ratios, and use only the luminances. Thus, the previous result for the Mach card is in fact correct.

A variation on the Mach card that I found works even better than the original version is to do it the other way around. That is, to place the card as an inside-corner facing the observer, and perceptually flip it so it looks like an open book with the pages face-down. This “backwards” Mach card is included as Card 3. The observer then sees the following:
Region 2 is the lightest, since that is the only place where light from the light source can directly reach the card. Region 3 is the darkest because it is where region 1 casts a shadow. Region 1 is slightly lighter than region 3 because a small amount of the light is able to shine through the card.

One reason why this works better than the original illusion could be because region 2 is white, while the rest of the card (even the rest of the side facing the light source) is darker. According to Rock, “what we call ‘white’ … is essentially … regions with the highest … possible relative reflectance value” (Rock 210 - 211). Thus, it is possible that the presence of the white region makes everything else look darker simply by being brighter than everything else. It makes everything else look dark in comparison.

However, if this were the case, then if we cut out region 2, the illusion should be lessened. Card 4 has exactly this region cut out. But (at least for me), the illusion for Card 4 is no less intense than it is for Card 3. In addition, if a white comparison surface is introduced into the scene after Card 4 is flipped, the illusion does not become any more intense. Thus, I do not believe that region 2 being white is what is responsible for the strength of the illusion.

A better explanation uses the fact that the luminance of region 3 is lower than the luminance anywhere in the original Mach card (that is, Card 1 or 2). Because differences in luminance between sides of a dihedral angle tend to be attributed to illumination
differences and not reflectance differences, an observer tends to process the color (i.e. reflectance) of the card to be the same for regions 1 and 3. Once the card is flipped, there is no shadow perceived in region 3, since region 1 is not in a position to cast a shadow. This means the perceived difference in illumination between regions 1 and 3 is lessened. So region 1 must become very dark in order to have the same reflectance as region 3.

The Mach card illusion demonstrates that the processes of depth perception and brightness constancy are very much connected in the real three-dimensional world. When an observer perceptually reverses the orientation of the Mach card, he/she does not perceive the true color of the card. But when the observer looks at the card normally with both eyes, he/she easily perceives the card’s real color. The difference in perceived color between the two scenarios is due to a difference in the perceived three-dimensional form of the card. Thus, there is a strong connection between depth perception and brightness constancy visual processes.
References


